dual fields represent a plane wave propagating in the same direction as the original wave, however its transverse fields have been rotated by an angle $\frac{\rho\pi}{2}$. By substituting γ, one gets the complete forms of the fractional dual fields including all fractional parameters:

$$E_{fd} = E_0\left[cos\left(\frac{\rho\pi}{2}\right)\hat{x} + sin\left(\frac{\rho\pi}{2}\right)\hat{y}\right] e^{-\left(\sqrt{\mu\epsilon}(\omega)^{\frac{\alpha+\beta}{2}} cos\left(\frac{\alpha+\beta}{4}\pi\right)\right)z} e^{i\left(\sqrt{\mu\epsilon}(\omega)^{\frac{\alpha+\beta}{2}} sin\left(\frac{\alpha+\beta}{4}\pi\right)\right)z} \quad (20a)$$

$$\eta_f H_{fd} = E_0\left[-sin\left(\frac{\rho\pi}{2}\right)\hat{x} + cos\left(\frac{\rho\pi}{2}\right)\hat{y}\right] e^{-\left(\sqrt{\mu\epsilon}(\omega)^{\frac{\alpha+\beta}{2}} cos\left(\frac{\alpha+\beta}{4}\pi\right)\right)z} e^{i\left(\sqrt{\mu\epsilon}(\omega)^{\frac{\alpha+\beta}{2}} sin\left(\frac{\alpha+\beta}{4}\pi\right)\right)z} \quad (20b)$$

The last form indicates that taking into account fractional time derivatives a general loss term is shown to indicate losses observed upon wave propagation in free-source mediums and additional field solutions intermediate to the dual ones can be obtained by considering fractional curl operators.

Certainly, it's more convenient to have a practical example from the conventional case to see the effect of imposing fractional parameters on its performance. The rectangular waveguide case is selected and studied as a point of future work.

IV. Conclusions

Maxwell's curl equations are reconsidered replacing the integer-order time derivatives by fractional ones. By applying the modified form on the operation of an electromagnetic plane wave propagating in a source-free medium a general expression for the loss term of the wave is introduced which recovers the conventional case by setting all fractional derivatives to unity. Afterwards, the modified formulas of the fractional curl operator taking into account the modified form of Maxwell's curl equations are introduced. Following this, an additional degree of freedom to control the characteristics of the fractional dual solutions is introduced due to the extra fractional parameter. Applying this work on a practical example of a rectangular waveguide is a point of future work expected to introduce significant results.

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