

dual fields represent a plane wave propagating in the same direction as the original wave, however its transverse fields have been rotated by an angle $\frac{\rho\pi}{2}$. By substituting γ , one gets the complete forms of the fractional dual fields including all fractional parameters:

$$E_{fd} = E_0 \left[\cos\left(\frac{\rho\pi}{2}\right) \hat{x} + \sin\left(\frac{\rho\pi}{2}\right) \hat{y} \right] e^{-\left(\sqrt{\mu\epsilon}(\omega)^{\frac{\alpha+\beta}{2}} \cos\left(\frac{\alpha+\beta}{4}\pi\right)\right)z} e^{i\left(\sqrt{\mu\epsilon}(\omega)^{\frac{\alpha+\beta}{2}} \sin\left(\frac{\alpha+\beta}{4}\pi\right)\right)z} \quad (20a)$$

$$\eta_f H_{fd} = E_0 \left[-\sin\left(\frac{\rho\pi}{2}\right) \hat{x} + \cos\left(\frac{\rho\pi}{2}\right) \hat{y} \right] e^{-\left(\sqrt{\mu\epsilon}(\omega)^{\frac{\alpha+\beta}{2}} \cos\left(\frac{\alpha+\beta}{4}\pi\right)\right)z} e^{i\left(\sqrt{\mu\epsilon}(\omega)^{\frac{\alpha+\beta}{2}} \sin\left(\frac{\alpha+\beta}{4}\pi\right)\right)z} \quad (20b)$$

The last form indicates that taking into account fractional time derivatives a general loss term is shown to indicate losses observed upon wave propagation in free-source mediums and additional field solutions intermediate to the dual ones can be obtained by considering fractional curl operators.

Certainly, it's more convenient to have a practical example from the conventional case to see the effect of imposing fractional parameters on its performance. The rectangular waveguide case is selected and studied as a point of future work.

IV. CONCLUSIONS

Maxwell's curl equations are reconsidered replacing the integer-order time derivatives by fractional ones. By applying the modified form on the operation of an electromagnetic plane wave propagating in a source-free medium a general expression for the loss term of the wave is introduced which recovers the conventional case by setting all fractional derivatives to unity. Afterwards, the modified formulas of the fractional curl operator taking into account the modified form of Maxwell's curl equations are introduced. Following this, an additional degree of freedom to control the characteristics of the fractional dual solutions is introduced due to the extra fractional parameter. Applying this work on a practical example of a rectangular waveguide is a point of future work expected to introduce significant results.

REFERENCES

- [1] N. Engheta, "Use of Fractional Integration to Propose Some "Fractional" Solutions for the Scalar Helmholtz Equation," Progress In Electromagnetics Research, PIER 12, 107-132, 1996.
- [2] Q. A. Naqvi and A. A. Rizvi, "Fractional Solutions for the Helmholtz's Equation in a Multilayered Geometry," Progress In Electromagnetics Research, PIER 21, 319-335, 1999.
<http://dx.doi.org/10.2528/PIER98100501>
- [3] M. Zubair, M. J. Mughal, Q. A. Naqvi, and A. A. Rizvi, "Differential Electromagnetic Equations in Fractional Space," Progress In Electromagnetics Research, PIER 114, 255-269, 2011
<http://dx.doi.org/10.2528/PIER11011403>
- [4] M. Zubair, M. J. Mughal, and Q. A. Naqvi, "An Exact Solution of the Cylindrical Wave Equation for Electromagnetic Field in Fractional Dimensional Space," Progress In Electromagnetics Research, PIER 114, 443-455, 2011.
<http://dx.doi.org/10.2528/PIER11021508>
- [5] A. Shamim, A. G. Radwan, K. N. Salama, "Fractional Smith Chart Theory and Application," IEEE Microwave and Wireless Components Letters, vol. 21, 3, 117- 119, 2011.
<http://dx.doi.org/10.1109/LMWC.2010.2098861>

- [6] A. G. Radwan, A. Shamim, K. N. Salama, "Theory of Fractional-order Elements Based Impedance Matching Networks," IEEE Microwave and Wireless Components Letters, vol. 21, 3, 120- 122, 2011.
<http://dx.doi.org/10.1109/LMWC.2010.2103051>
- [7] S. G. Samko, A. A. Kilbas and O. I. Marichev, Fractional integrals and derivatives: theory and application, Gordon & Breach, 1987.
- [8] J. Sabatier, O.P. Agrawal and J.A. Tenreiro Machado, Advances in fractional calculus; theoretical developments and applications in physics and engineering, Springer, 2007.
<http://dx.doi.org/10.1007/978-1-4020-6042-7>
- [9] R. L. Magin, Fractional calculus in bioengineering, Begell House, Connecticut, 2006.
- [10] R. Martin, J. J. Quintara, A. Ramos and L. De La Nuez, "Modeling electrochemical double layer capacitor, from classical to fractional impedance," Journal of Computational and Nonlinear Dynamics, vol. 3, no. 2, 2008.
<http://dx.doi.org/10.1115/1.2833909>
- [11] A. G. Radwan, A. S. Elwakil, and A. M. Soliman, "Fractional-order Sinusoidal Oscillators: Design Procedure and Practical Examples," IEEE Trans. Circuits & Syst.-I, vol. 55, pp. 2051-2063, 2008.
- [12] F. Mainardi and P. Paradisi, "Model of diffusive waves in viscoelasticity based on fractional calculus," in proceedings of the IEEE Conference on Decision and control, vol. 5, O.R. Gonzales, IEEE, New York, . 4961-4966, 1997.
<http://dx.doi.org/10.1109/CDC.1997.649833>
- [13] R. Metzler and J. Klafter, "Boundary value problems for fractional diffusion equations", Physica A 278,107-125,2000.
[http://dx.doi.org/10.1016/S0378-4371\(99\)00503-8](http://dx.doi.org/10.1016/S0378-4371(99)00503-8)
- [14] R. Ismail and A.G. Radwan, "Rectangular waveguides in the fractional-order domain", in proceedings of the 1st international conference of engineering and technology(ICET 2012), IEEE, ,Egypt, 2012.
<http://dx.doi.org/10.1109/icengtechnol.2012.6396151>
- [15] N. Engheta, "Fractional curl operator in electromagnetics," Microwave and Optical Technology Letters," Vol. 17, No. 2, 86-91, 1998.
[http://dx.doi.org/10.1002/\(SICI\)1098-2760\(19980205\)17:2<86::AID-MOP4>3.3.CO;2-Z](http://dx.doi.org/10.1002/(SICI)1098-2760(19980205)17:2<86::AID-MOP4>3.3.CO;2-Z)
- [16] A. Hussain, S. Ishfaq, and Q. A. Naqvi, "Fractional Curl Operator and Fractional Waveguides," Progress In Electromagnetics Research, PIER 63, 319-335, 2006.
<http://dx.doi.org/10.2528/PIER06060604>
- [17] M. Faryad and Q. A. Naqvi , "Fractional Rectangular Waveguide," Progress In Electromagnetics Research, PIER 75, 383-396, 2007
<http://dx.doi.org/10.2528/PIER07052803>
- [18] D.M. Pozar, Microwave Engineerin, 3rd edition, John Wiley & Sons, 2004.
- [19] M. Nakagawa, and K. Sorimachi, "Basic characteristics of a fractance device," IEICE Trans. Fundam. Electron. Commun. Comput. Sci., vol. E75, no.12, pp. 1814-1819, 1992.
- [20] Modeling Coilcraft RF Inductors. Coilcraft Document 158 May 1999 [Online]. Available: <http://www.coilcraft.com>.
- [21] M.N.O. Sadiku, Elements of Electro-magnetics, 5th edition, Oxford University Press, 2010.



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