

Off-Road Vehicle Suspension Performance Improvement Using Linear Quadratic Regulator Techniques

Musa M. Bello, Amir A. Shafie, and Raisuddin Md. Khan

Abstract—The main purpose of vehicle suspension system is to isolate the vehicle main body from any road geometrical irregularity in order to improve the passengers ride comfort and to maintain good handling stability. The present work aim at designing a control system for an active suspension system which is widely applied in today's automotive industries. The design implementation involves construction of a state space model for half vehicle with four degree of freedom and a development of full state-feedback controller using Linear Quadratic Regulator (LQR) techniques. The performance of active suspension system was assessed by comparing its response to that of passive suspension system. Simulation using Matlab/Simulink environment shows that, even at resonant frequency the active suspension system develops a good dynamic response and a better ride comfort when compared with the passive suspension system.

Index Terms—Active suspension, half car, LQR control, Matlab/Simulink.

I. INTRODUCTION

It is a clearly known fact that researchers everywhere in the world are constantly developing interest in attaining a satisfactory system performances in vehicles suspension system for the past two decades [1, 2]. Vehicle suspension systems support the vehicle weight by providing an effective isolation of the vehicle body from road irregularities, by keeping and maintaining the wheels contact in appropriate position with the road surface [3]. Suspension system in vehicles play a significant role in ensuring the stability of the system and the performances are making better. It is quite important that the problem of control design should be given a great consideration for active suspension systems.

Suspension systems can be classified into Passive, Semi-Active and Active suspension systems [4]. Passive vibration control, known as passive suspension is the commonly used conventional suspension system of linear spring and viscous damper with constant stiffness and damping coefficients. This fixed stiffness and damping coefficients characteristic causes the passive suspension to be of classic compromise between vehicle ride comfort and

vehicle road handling [4, 5].

Semi-active suspension allows smooth changing of passive damper with a semi-active damping coefficient; it can be closely as efficient as full active suspension in improving ride quality and comfort. Magneto-rheological and Electro-rheological fluid dampers are a good examples of semi-active suspension device [6, 7]. These dampers possessed the ability of changing damping coefficient accordingly.

Active suspensions differ from the conventional passive suspensions in their ability to interject energy into the system, as well as store and dissipate this energy. This type of suspension system contains the power controlled actuator in place of passive damper which is placed in between the wheels and vehicle body; these actuators can be electric motor, pneumatic or hydraulic actuator [8]. Compared to passive suspension systems, active suspensions improve the performance of the suspension system over a wide range of frequencies.

Subsequently, active suspension systems can give what it is desired or needed for the passengers comfort and ensure other suspension performances by sufficing the basic purpose of isolating passengers from road-induced shocks and vibration [9]. Hence, a problem of active suspension systems control design is considered as an important research topic for accomplishing the desired vehicle suspension operations.

From the comprehensive literature study, it has been discovered that active suspension system can sustained a good driving comfort and durability of vehicle for versatile operating considerations [10-13]. Attention of numerous study research's and automotive industries in the field of active suspension system was attracted because of its advanced feature characteristics which essentially acquire a victory over the limitations of passive suspension system [14, 15].

The present research work efforts to study the performance of an active control strategy in the application of a model that involves four degree of freedom vehicle suspension and developing of a full state-feedback controller using Linear Quadratic Regulator (LQR) approach, vehicle dynamic response with road uncertainties simulated with fixed parameters of the system.

II. PROBLEM STATEMENT

In active suspension system control, the required performance is based on the following

A. Ride comfort

This is referred to as vehicle body motion felt by

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passengers and can be measured by both the vertical and pitch motion of the vehicle body in the case of a half car model. The main objective for active suspension system is to design a control system that will stabilize and isolate the vibratory forces transferred from the axle to the car body.

B. Suspension travels

This referred to the relative displacement between the vehicle body and the wheel. This has a good performance when the rattle space between the vehicle body and wheel are kept very small. Due to mechanical structure, the suspension travel should not outmatch the allowable maximum, which is described as $|z_{si} - z_{ui}| \leq z_{i,max}$, where i is either front (f) or rear (r).

C. Road handling

Good road handling is classified with the forces between the road surface and the wheels. Dynamic tire load should not outmatch the static tire load for both the front and rear wheels, which can be described as;

$$|k_{sf}(z_{uf} - z_{rf})| \leq F_f, \quad |k_{sr}(z_{ur} - z_{rr})| \leq F_r$$

Where,

$$F_r = \frac{m_s g l_1 + m_{ur} g (l_1 + l_2)}{l_1 + l_2} \quad \text{And}$$

$$F_f = (m_s + m_{uf} + m_{ur}) g - F_r$$

F_f And F_r Denotes the front and rear static tire loads respectively.

III. SYSTEM MATHEMATICAL MODEL

The main objective for obtaining the mathematical model of any system is to know the dynamic model of the system, and also described the relationship between the input and output and enables one to understand the behavior of the system.

The four degree of freedom half vehicle model for active suspension systems used in this work is shown in fig 1. The passive system model can be obtained by setting $u_f = u_r = 0$.

In the half car model, only the vertical and pitch angular acceleration of the vehicle body is considered. The effect of roll motion is neglected. From Fig. 1, m_s and I represent the body sprung mass and mass moment of inertia for pitch motion respectively. m_{uf} and m_{ur} the front and rear unsprung masses, $F_{ksf}, F_{ksr}, F_{csf}, F_{csr}$ represent the front and rear suspension forces by the springs and dampers respectively. F_f, F_r implies the front and rear tire forces respectively.

The vehicle body vertical displacement at the centre of gravity is z_c . θ is the pitch angular displacement, z_{uf}, z_{ur} are the unsprung mass vertical displacements respectively. z_{rf}, z_{rr} are the road inputs. l_1, l_2 Represent the suspension distances from the centre of the vehicle sprung mass, and u_f, u_r are the control inputs actuator forces for

front and rear suspensions respectively.

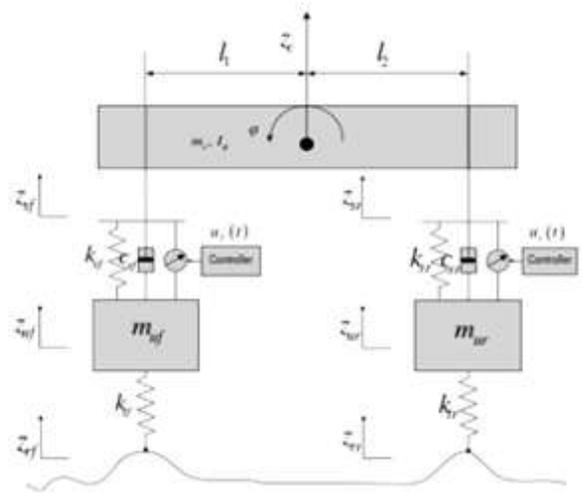


Fig. 1 Half Vehicle Model

By using either Lagrange or Newton's second law of motion, the dynamics equation of motion for the model can be obtained as;

$$m_s \ddot{z}_c = F_{ksf} + F_{csf} + F_{ksr} + F_{csr} + u_f + u_r \quad (3.1)$$

$$I_\theta \ddot{\theta} = -l_1 (F_{ksf} + F_{csf} + u_f) + l_2 (F_{ksr} + F_{csr} + u_r) \quad (3.2)$$

$$m_{ur} \ddot{z}_{ur} = F_{ktr} - F_{ksr} - F_{csr} - u_r \quad (3.3)$$

$$m_{uf} \ddot{z}_{uf} = F_{krf} - F_{ksf} - F_{csf} - u_f \quad (3.4)$$

Where,

$$F_{ksf} = k_{sf} (z_{uf} - z_{sf}), \quad F_{ksr} = k_{sr} (z_{ur} - z_{sr}) \quad (3.5)$$

$$F_{csf} = c_{sf} (\dot{z}_{uf} - \dot{z}_{sf}), \quad F_{csr} = c_{sr} (\dot{z}_{ur} - \dot{z}_{sr}) \quad (3.6)$$

$$F_{krf} = k_{rf} (z_{uf} - z_{rf}), \quad F_{ktr} = k_{tr} (z_{ur} - z_{rr}) \quad (3.7)$$

$$z_{sf} = z_c - l_1 \theta, \quad z_{sr} = z_c + l_2 \theta \quad (3.8)$$

$\ddot{z}_c, \ddot{z}_{uf}$ and \ddot{z}_{ur} are the vertical accelerations of the sprung mass centre of gravity, front and rear unsprung masses respectively. $\ddot{\theta}$ is the body pitch angular acceleration.

The state space equation representation of the half vehicle model is

$$\dot{x}(t) = Ax(t) + Bu(t) + Gr(t) \quad (3.9)$$

$$y = Cx(t) + Du(t) \quad (3.10)$$

Let the state variables are;

$$x_1 = z_{sf} - z_{uf}, \quad x_2 = z_{sr} - z_{ur}, \quad x_3 = z_{uf} - z_{rf},$$

$$x_4 = z_{ur} - z_{rr}, \quad x_5 = \dot{z}_{sf}, \quad x_6 = \dot{z}_{sr}, \quad x_7 = \dot{z}_{uf}, \quad x_8 = \dot{z}_{ur}$$

$$x(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t) \quad x_5(t) \quad x_6(t) \quad x_7(t) \quad x_8(t)]^T$$

$$u(t) = [u_f(t) \quad u_r(t)]^T, \quad r(t) = [\dot{z}_{rf}(t) \quad \dot{z}_{rr}(t)]^T$$

The matrices A, B, G, C and D are;

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -a_1k_{sf} & -a_2k_{sr} & 0 & 0 & -a_1c_{sf} & -a_2c_{sr} & a_1c_{sf} & a_2c_{sr} \\ -a_2k_{sf} & -a_2k_{sr} & 0 & 0 & -a_2c_{sf} & -a_2c_{sr} & a_2c_{sf} & a_2c_{sr} \\ \frac{k_{sf}}{m_{uf}} & 0 & \frac{-k_{sf}}{m_{uf}} & 0 & \frac{c_{sf}}{m_{uf}} & 0 & \frac{-c_{sf}}{m_{uf}} & 0 \\ 0 & \frac{k_{sr}}{m_{ur}} & 0 & \frac{-k_{sr}}{m_{ur}} & 0 & \frac{c_{sr}}{m_{ur}} & 0 & \frac{-c_{sr}}{m_{ur}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & a_1 & a_2 & \frac{-1}{m_{uf}} & 0 \\ 0 & 0 & 0 & 0 & a_2 & a_3 & 0 & \frac{-1}{m_{ur}} \end{bmatrix}^T$$

$$G = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, D = 0$$

Where,

$$a_1 = \left(\frac{1}{m_s} + \frac{l_1^2}{I_\theta} \right), a_2 = \left(\frac{1}{m_s} + \frac{l_1 l_2}{I_\theta} \right), a_3 = \left(\frac{1}{m_s} + \frac{l_2^2}{I_\theta} \right)$$

The state variables of interest are the sprung mass vertical acceleration \ddot{z}_c , sprung mass pitch angular acceleration $\ddot{\theta}$, front and rear vehicle suspension deflections $(z_{sf} - z_{uf})$ and $(z_{sr} - z_{ur})$ respectively, and the front and rear tire deflections $(z_{uf} - z_{rf})$ and $(z_{ur} - z_{rr})$ respectively.

IV. ROAD INPUT DISTURBANCE

Road surface is believed to be a natural changing condition as well as the major cause of input disturbance when dealing with vehicle suspension systems. For a better comfort during riding, it is necessary to have a perfect road surface model to design an active vehicle suspension control system. Road inputs analytically can be distinguished in many possible ways, which can be classified either as shock or vibration [11, 16].

In this work, a discrete type of road input disturbance was used and is commonly classified as a shock induced road input disturbance due to its ability to convey an impact forces to the vehicle within a short period of time. Examples include pronounced bumps, potholes, steps etc [17]. One of the examples which is speed bumps are used to impel most vehicle drivers in order to reduce the vehicle speed to levels off within the speed limits of a specific place such as residential area, schools, markets etc.

The front and rear wheel input disturbances, z_{rf} and z_{rr} respectively, are expressly shown in (4.1) and (4.2) for which a denotes the bump amplitude, t denotes the simulation time in seconds, t_d denotes the delay time between the front and

rear wheels, L represent the disturbance wavelength and finally V stands for the vehicle forward velocity.

$$z_{rf} = \begin{cases} \frac{a}{2} (1 - \cos(\frac{2\pi V}{L} t_f)), 0 \leq t_f \leq \frac{L}{V} \\ 0, otherwise \end{cases} \quad (4.1)$$

$$z_{rr} = \begin{cases} \frac{a}{2} (1 - \cos(\frac{2\pi V}{L} t_r)), t_d \leq t_r \leq t_d + \frac{L}{V} \\ 0, otherwise \end{cases} \quad (4.2)$$

Where,

$$t_d = \frac{(l_1 + l_2)}{V} \quad (4.3)$$

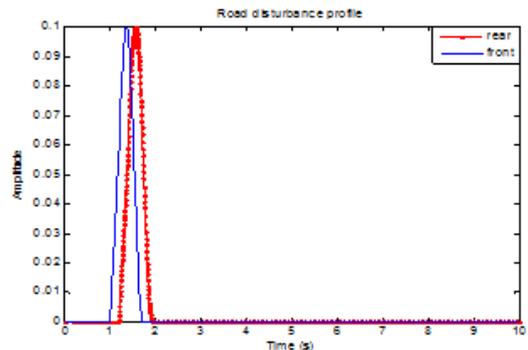


Fig. 2 Road Input Disturbance

V. CONTROLLER DESIGN

The control system design for the half vehicle active suspension was based on Linear Quadratic Regulator (LQR) method. For the purpose of this work, we assumed that all the states for the system described in (3.9) above can be measured.

The control system was designed to minimized the output weighted performance index

$$J = \frac{1}{2} \int_0^\infty (x^T Q x + u^T R u) dt \quad (5.1)$$

Where Q is a symmetric positive semi definite and R is positive definite weighted matrices. The definiteness assumptions of both Q and R assure that J is a non-negative and contributes to a sensible minimization performance.

The minimized performance index solution of the LQR is a feedback law that states

$$u = -Kx \quad (5.2)$$

Where K is a feedback gain that can be obtained from the Algebraic Riccati Equation (ARE)

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \quad (5.3)$$

Where,

$$K = R^{-1}B^T P \quad (5.4)$$

Equation (3.9) becomes

$$\dot{x}(t) = (A - BK)x(t) + Gr(t) \quad (5.5)$$

Matlab/Simulink design optimization tool box was used to obtain the values for the element of Q and R matrices used.

$$Q = \text{diag}([5e001 \ 1.1e002 \ 1e002 \ 0 \ 1.022e002 \ 0 \ 5.042e001 \ 5.2e002]) \quad (5.6)$$

$$R = \text{diag}([3.2e00 - 2 \ 1.32e00 - 2]) \quad (5.7)$$

VI. SIMULATION RESULT AND DISCUSSION

The half vehicle suspension system model parameters value used for the simulation is listed in table 1.

TABLE I
PARAMETERS USED FOR THE HALF VEHICLE MODEL

Symbols	Parameters	Values
m_s	Sprung mass	730 (kg)
I_θ	Pitch moment of inertia	2460 (kgm ²)
m_{uf}	Front unsprung mass	40 (kg)
m_{ur}	Rear unsprung mass	35.5 (kg)
k_{sf}	Front suspension stiffness	19,960 (N/m)
k_{sr}	Rear suspension stiffness	17,500 (N/m)
c_{sf}	Front suspension damping coefficient	1290 (Ns/m)
c_{sr}	Rear suspension damping coefficient	1620 (Ns/m)
k_{tf}	Front tire spring stiffness	175,500 (N/m)
k_{tr}	Rear tire spring stiffness	175,500 (N/m)
l_1	Distance from m_s C.G to front axle	1.011 (m)
l_2	Distance from m_s C.G to rear axle	1.803 (m)
u_f	Front actuator force	-
u_r	Rear actuator force	-
V	Forward velocity	12.5 (m/s)
a	Amplitude	10 (cm)
L	Wave length	9.1 (m)

Transient responses of both passive and active suspension system were determined in time domain analysis for a sinusoidal road input profile. The sinusoidal input characterizes a car moving on a road that is having a succession of waves or curves with amplitude of 10 cm, a wavelength of 9.1 m and forward velocity of 45 km/hr. Matlab/Simulink environment was used to build the model.

Fig. 3 and 4 show the suspension deflection time histories for the front and rear wheels of both active and passive suspensions. It's clearly shown that the maximum suspension travel for the wheels at the maximum height of the road disruption input and there values were less than the defined suspension travel limits of 0.1 m. The input disturbance effect was entirely oppressed after about 3.0s and 2.8s for front and rear active suspension whereas it is 5.0s and 4.2s for front and rear passive suspension respectively.

The active system was able to reduce the disturbance by suppressing it to about 2.0s in fig. 5 below when compared to about 3.2s of the passive system. The sprung mass acceleration deviate from -2.4 m/s² to 6.0 m/s² for the active system, whereas, it is between -2.7 m/s² to 6.2 m/s² for passive system which is within the ISO 2631 classified condition range of fairly uncomfortable for occupants in public transport [15, 18].

Fig. 6 described the pitch angular acceleration of the vehicle with magnitude ranging between -0.6 rad/s² and 0.9 rad/s² for active suspension, whereas, it is between -1.5 rad/s² and 1.6 rad/s² for passive suspension system respectively. This gave about 44% improvement in the passengers comfort compared to the passive system.

The vehicle road handling time history was shown in fig. 7 and 8 for both front and rear wheels respectively. The uttermost road holding capability value of 0.007 m and 0.01 m was attained for front active and passive wheels respectively. Whereas, 0.006 m and 0.0085 m was attained for

rear active and passive wheels respectively. More so, these obtained values were less than the maximum allowable values of 0.1 m for road handling capacity put forward in this work.

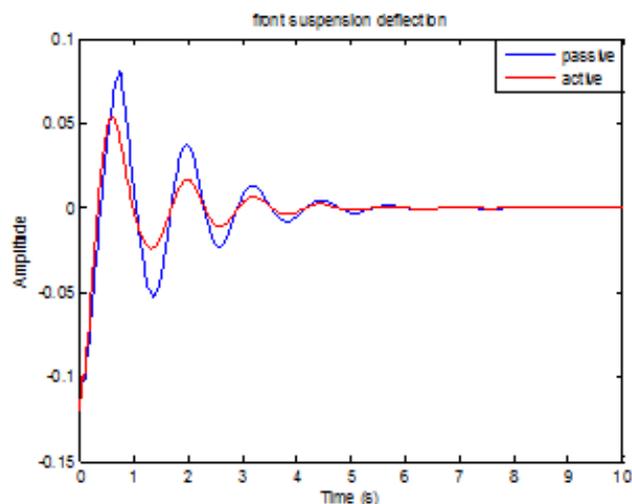


Fig. 3 Front Suspension Travel

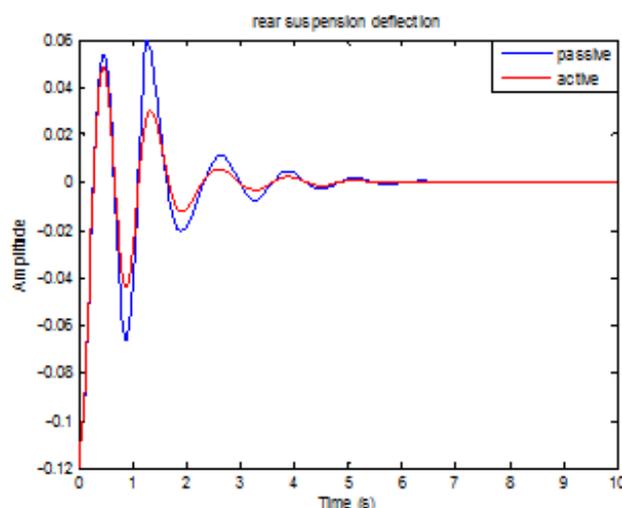


Fig. 4 Rear Suspension Travel

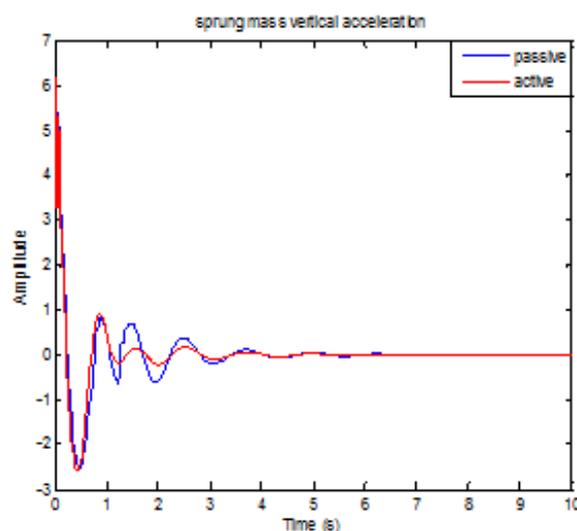


Fig. 5 Sprung Mass Vertical Acceleration

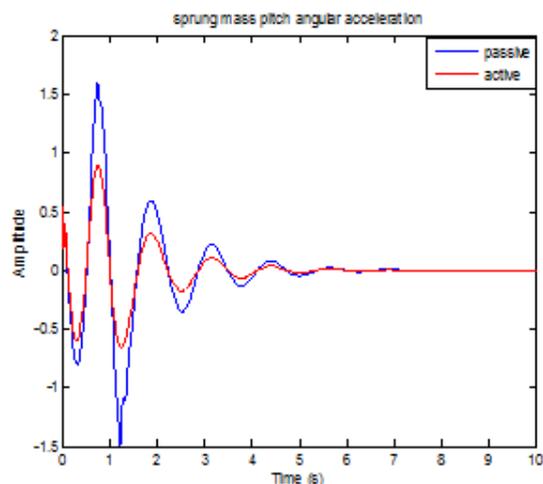


Fig. 6 Vehicle Pitch Angular Acceleration

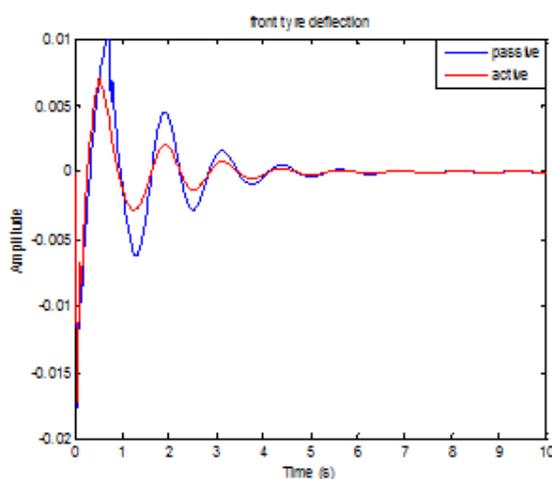


Fig. 7 Front Wheel Deflection

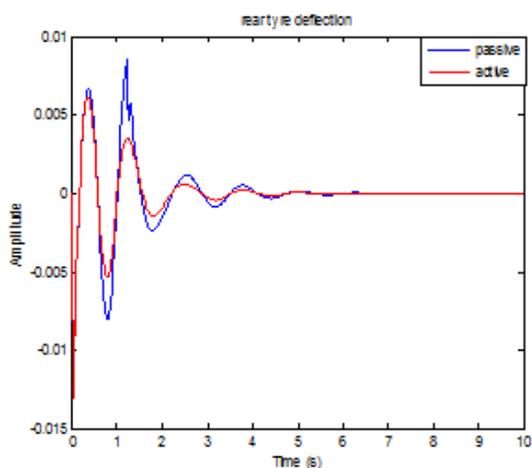


Fig. 8 Rear Wheel Deflection

VII. CONCLUSION

In this paper the performance of both passive and active suspension system were analyzed carefully. LQR controller was designed for a half vehicle model in order to improve the vehicle performance. The controller configuration was simulated and verified for the model; comparison of simulated results showed that LQR based active suspension system has a better passenger ride comfort and a good dynamic response than the conventional passive suspension system. For a sinusoidal bump input of 10 cm amplitude and

9.1 m wavelength, the sprung mass vertical acceleration and the pitch angular acceleration has been reduced by 4.8% and 43.8% respectively which indicates an improvement in the ride comfort, also the suspension travel for both front and rear wheels has been reduced by 32% and 19% respectively which shows a reduction in the suspension rattle space. At the same time, the road handling capacity was improved for both front and rear wheels by 30% and 29.4% respective. Hence, from the above result we come to the conclusion that despite the unavoidably challenges of power consumption and cost effect of the active suspension system, it has a better performance potentialities when compared to the conventional passive suspension system.

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